1. What is the product of all valid solutions of the fractional equation $\dfrac{x}{45-5x} = \dfrac{1}{5(x-1)}$?

(a) -9 (b) -3 (c) 0 (d) 3 (e) None of these

Answer: (a)

2. Use the statement below to answer the question that follows.

If a quadrilateral is not a rhombus, then it is not a square.

If the statement above is true, then which of the following statements must be true?

(a) If a quadrilateral is not a square, then it is a rhombus.

(b) If a quadrilateral is a square, then it is a rhombus.

(c) If a quadrilateral is a rhombus, then it is a square.

(d) If a quadrilateral is not a square, then it is not a rhombus.

(e) If a quadrilateral is a rhombus, then it is not a square.

Answer: (b)

3. Find the value of $\dfrac{5}{3} + 3\dfrac{1}{4} - \dfrac{11}{12}$.

(a) $-2\dfrac{1}{3}$ (b) 1 (c) $2\dfrac{3}{4}$ (d) 4 (e) $5\dfrac{1}{12}$

Answer: (d)

4. If three numbers are randomly selected without replacement from 0, 1, 2, 3, and 4, how many three digit numbers can we make?

(a) 125   (b) 64   (c) 60   (d) 48   (e) 24

Answer: (d)

5. The graph of a function $f(x) = \dfrac{a}{x^2}$ passes through two points $(8, 2)$ and $(b, 2b)$. Find the sum of a and b.

(a) 4   (b) 32   (c) 68   (d) 128   (e) 132

Answer: (e)

6. If Brittany gets a 97 on her next math test, her average will be 89. However, if she gets 72, her average will be 84. How many tests has Brittany already taken?

(a) 3   (b) 4   (c) 5   (d) 7   (e) 8

Answer: (b)

7. We partition the rectangle $ABCD$ into seven congruent rectangles like the figure below. If the area of the $\square ABCD$ is 84 cm^2, what is the perimeter of a small rectangle in cm? (*Figure not drawn to scale!*)
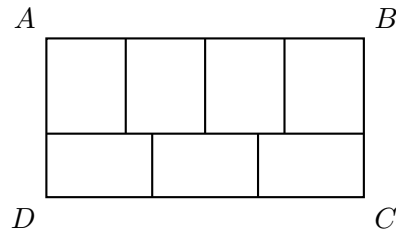
(a) 9   (b) 7   (c) 14   (d) 19   (e) 38

Answer: (c)

8. Tom plants 5 white rosebushes and 2 red rosebushes in a row. How many different ways can he plant at least four white bushes consecutively?

(a) 21   (b) 4   (c) 6   (d) 9   (e) 12

Answer: (d)

9. Two circles, A and B, overlap each other as shown below. The area of the common part is $\dfrac{2}{5}$ of the area of Circle A, and $\dfrac{5}{8}$ of the area of Circle B. What is the ratio of the radius of Circle A to that of Circle B? (*Figure not drawn to scale!*)
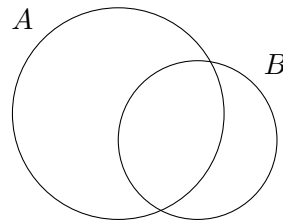
(a) $2:1$ (b) $3:2$ (c) $4:3$ (d) $5:4$ (e) $6:5$

Answer: (d)

10. Six students take a photo for a yearbook sitting in a row. Two of them wear hats, and they want to sit at both ends. How many different ways of seating all the students are possible?

(a) 24   (b) 48   (c) 60   (d) 120   (e) 1440

Answer: (b)

11. Increasing a number by 3.25% is equivalent to multiplying it by which of the following?

(a) 1.325   (b) 1.0325   (c) 1.00325   (d) 0.325   (e) 0.0325

Answer: (b)

12. Let m be the average of numbers x, y, and z. Find the average of numbers, $\frac{1}{b}(3x - m)$, $\frac{1}{b}(3y - m)$, and $\frac{1}{b}(3z - m)$, where b is a non-zero real number.

(a) $\frac{2m}{3}$ (b) m (c) $\frac{2m}{b}$ (d) $\frac{m}{b}$ (e) $-\frac{m}{b}$

Answer: (c)

13. The greatest common divisor of two possible numbers is 14 and the least common multiple is 518. Find the sum of the two numbers.

(a) 38   (b) 51   (c) 378   (d) 518   (e) 532

Answer: (e)

14. Martha's parents presented her a saving account with the balance of \$81,000 on her birthday this year. When she was born, her parents deposited \$3,000, which tripled every four years. How old is Martha?

(a) 4 years old   (b) 8 years old   (c) 12 years old   (d) 16 years old   (e) 27 years old

Answer: (c)

15. Let A and B be natural numbers such that $\frac{A}{2} = \frac{B}{3}$. Let M be the least common multiple of A and B, and let D be the greatest common divisor of A and B. If the product of M and D is 216, then find $B - A$.

(a) 6   (b) 12   (c) 18   (d) -6 (e) -12

Answer: (a)

16. A town has a basketball team and a football team, and both teams play once a week. The basketball team wins 1 out of 3 games, and the football team loses 2 out of 5 games. What is the probability that both teams win on the same week?

(a) $\frac{11}{15}$ (b) $\frac{14}{15}$ (c) $\frac{3}{8}$ (d) $\frac{2}{15}$ (e) $\frac{1}{5}$

Answer: (e)

17. Solve the equation $3x - [5 - 3(x + 2)] = 4$.

(a) -1 (b) $\frac{1}{2}$ (c) $\frac{5}{6}$ (d) 3   (e) 5

Answer: (b)

18. Jamie buys a 59 inch long sub sandwich to share with his brothers. He cuts it into three pieces so that the mid-sized piece is 5 inch longer than the shortest piece and the shortest piece is 9 inch shorter than the longest piece. How long in inches is the mid-sized piece?

(a) 15   (b) 17   (c) 20   (d) 24   (e) 25

Answer: (c)

19. In a middle school, there are 4 math classes. The average math scores of the classes are 79, 82, 84, and 75, and the number of students in each class is 30, 23, 21, and 26, respectively. Find the average score of the 4 math classes combined.

(a) 80   (b) 79   (c) 79.7   (d) 81.5   (e) 80.7

Answer: (c)

20. Let $x + 2y = 11$ and $3x + y = 13$. Find the value of $x + y$.

(a) 3   (b) 4   (c) 5   (d) 7   (e) 9

Answer: (d)

21. When the base of a triangle increases by 20% and the height to the base decreases by 10%, the area of the triangle will _____.

(a) increase by 10%   (b) decreased by 10%   (c) increase by 5%

(d) increase by 8%   (e) decrease by 8%

Answer: (d)

22. Let $a\nabla b = ab - a + b - 3$ for any integers a and b. Find the value of $2\nabla \left(\frac{1}{2}(3\nabla 5)\right)$.

(a) -63 (b) 16   (c) 10   (d) -15 (e) 19

Answer: (b)

23. Find the area bounded by $y = \frac{x}{2} + 1$ and the x-axis between $x = 3$ and $x = 7$.

(a) 8   (b) 10   (c) 14   (d) 28   (e) 45

Answer: (c)

24. Use the list of numbers below to answer the question that follows.

$$18, 5, 17, 12, k, 5, 9, 20$$

If the mean of the list of numbers above is 12, what is the median of the list?

(a) 10   (b) 10.5   (c) 11   (d) 11.5   (e) None of these

Answer: (c)

25. Which of the following is equivalent to the below expression?

$$\left(\frac{x^4}{y^2}\right)^3 \left(\frac{y^3}{x^6}\right)^5$$

(a) $x^{\frac{12}{30}} y^{\frac{15}{6}}$ (b) $\dfrac{y^9}{x^{18}}$ (c) $\dfrac{y^2}{x^4}$ (d) $\dfrac{y}{x^2}$ (e) None of these

Answer: (b)

26. Seven individual socks are drawn at random without replacement from a basket containing seven different pairs of socks. If the first six socks drawn are all different from one another, what is the probability that the next sock drawn at random from the basket is also different from the rest?

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{7}$ (d) $\dfrac{1}{8}$ (e) $\dfrac{1}{14}$

Answer: (b)

27. Find all value(s) of y which satisfy the following equations.
$$y = |1 - 2(x + 2)| \quad \text{and} \quad x^2 + 3x - 4 = 0$$

(a) 1   (b) 3   (c) 5   (d) 1, 3   (e) 3, 5

Answer: (c)

28. Which of the following number is the smallest when $p < -1$ and $0 < q < 1$?

(a) $-\dfrac{1}{p}$ (b) $-\dfrac{1}{q}$ (c) $|p|$ (d) $-p$ (e) $-q$

Answer: (b)

29. Amy and Bryan walk toward each other from A to B. Amy walks at 220 feet per minute and Bryan takes 15 minutes to walk a mile. The distance between A and B is 11 miles. If they start walking to each other at the same time, approximately how long will it take for them to meet?

(a) 40 minutes   (b) 1 hour   (c) 1 hour 20 minutes

(d) 1 hour 42 minutes   (e) 2 hours

Answer: (d)

30. Evaluate $1^2 - 2^2 + 3^2 - 4^2 + \cdots - 998^2 + 999^2$.

(a) 1,500,500   (b) 499,999   (c) 500,999   (d) 500,500   (e) 499,500

Answer: (e)

31. If it was two hours later, it would be half as long until midnight as it would be if it was an hour later. What time is it now?

(a) 7:00 PM   (b) 8:00 PM   (c) 9:00 PM   (d) 8:30 PM   (e) 9:30 PM
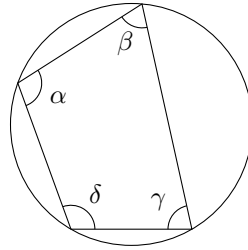
Answer: (c)

32. If x and y are distinct prime numbers, which of the following numbers must be odd?

(a) xy (b) $2(x + 1) - 2y + 1$ (c) $x + y + 3$

(d) $3xy - 1$ (e) $2x + y$

Answer: (b)

33. The interior angles of a quadrilateral inscribed in a circle are α, β, γ and δ as shown in the figure. Which of the following is always true?
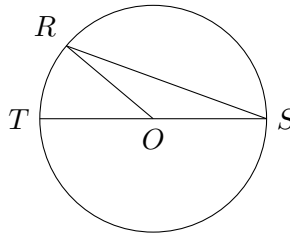

(a) $\alpha + \beta + \gamma + \delta = 180°$ (b) $\gamma + \delta = 90°$ (c) $\alpha + \beta = 90°$

(d) $\beta + \delta = 90°$ (e) $\alpha + \gamma = \beta + \delta$

Answer: (e)

34. Complete the following proof.

Consider a circle(center is O) that inscribes the triangle $\triangle RST$ and has the diameter $\overline{ST}$.

We want to show that $\angle RST = \dfrac{1}{2}\angle ROT$.

Proof: We begin by constructing radius $\overline{OR}$.

Since $\angle ROT$ is an exterior angle of $\triangle ROS$, $\angle ROT = $ (1).

Since the radius of the circle is $\overline{OR} = \overline{OS}$, $\angle ROS$ is (2).

Since $\triangle ROS$ is (2), $\angle ORS = $ (3).

Thus, $\angle RST = \dfrac{1}{2}\angle ROT$.

	(1)	(2)	(3)
(a)	$\angle ORS$	isosceles	$\angle OSR$
(b)	$\angle ORS + \angle OSR$	isosceles	$\angle OSR$
(c)	$\angle ORS + \angle OSR$	equiangular	$\angle OSR$
(d)	$\angle ORS$	isosceles	$\angle OSR$
(e)	$\angle ORS + \angle OSR$	isosceles	$\angle ROS$

Answer: (b)

35.   The Fine Art Center charged an admission fee of $5 per adult and $3 per child. On April 15, the center earned $1350 in total. On the next day, the Center earned $1365 and had the number of adults decreased by 5% and the number of children increased by 6%. How many children did the Center have on April 15?

(a) 90   (b) 114   (c) 120   (d) 250   (e) 265

Answer: (d)

36.   I had 1.5 gallons of 100% pure grape juice. After using 0.3 gallons of juice, I mixed 0.3 gallons of water with the remaining juice. Then I drank 0.3 gallons of the juice and mixed 0.3 gallons of water with the remaining juice. How pure is the remaining juice as a percentage?

(a) 45%   (b) 64%   (c) 75%   (d) 80%   (e) 92%

Answer: (b)

37.   In the table below, find the number in the 15th row and the 8th column.

$$
\begin{array}{llllll}
1 & 2 & 4 & 7 & 11 & \cdots \\
3 & 5 & 8 & 12 & \cdots \\
6 & 9 & 13 & \cdots \\
10 & 14 & \cdots \\
15 & \cdots \\
\vdots
\end{array}
$$

(a) 177   (b) 218   (c) 239   (d) 246   (e) 300

Answer: (d)

38.   There is a clock (which has two hands) in my classroom. After Mathematics class, I checked the time. The clock in the classroom indicated that is was 5:36 pm. If the hour hand moves continuously between 5 and 6, find the measure of the acute angle between two hands.

(a) 48°   (b) 52°   (c) 58°   (d) 66°   (e) 72°

Answer: (a)

39.   An urn contains 3 red and 5 black balls. Tom first withdraws a ball from the urn and Mary withdraws a ball from the urn without replacement. They keep withdrawing a ball alternately until a red ball is selected. Find the probability that Tom selects the red ball.

(a) $\frac{3}{8}$ (b) $\frac{15}{64}$ (c) $\frac{31}{56}$ (d) $\frac{48}{64}$ (e) $\frac{34}{56}$

Answer: (e)

40.   A lattice point is an ordered pair of integers (x, y). How many lattice points (including the end points) are there on the line segment joining the points $(4, 0)$ and $(18, 175)$?

(a) 7   (b) 8   (c) 9   (d) 12   (e) 13

Answer: (b)

41. Brian and Emma share the labor to build a hut. It would take 10 days to complete the hut if Brian works the first 6 days and then Emma works the rest 4 days after Brian. However, it would take 11 days if Brian works the first 9 days and then Emma works the rest 2 days after Brian. If Brian works alone, how many days will it take to complete the hut?

(a) 10 days   (b) 11 days   (c) 12 days   (d) 13 days   (e) 14 days

Answer: (c)

42. Tom read a book in 3 days. On the first day he read 1/5 of the book and 12 pages more. The next day he read 3/10 of what remained and 20 pages more. On the third day he read 3/4 of what remained and 30 pages more. How many pages were there in the book?

(a) 265   (b) 240   (c) 225   (d) 280   (e) 255

Answer: (a)

43. Sumin rolls three distinct dice and gets a, b, and c. Find the probability of $2a + b + c = 10$.

(a) $\frac{7}{18}$ (b) $\frac{7}{36}$ (c) $\frac{2}{27}$ (d) $\frac{7}{108}$ (e) $\frac{1}{18}$

Answer: (d)

44. Find the number of the ordered pairs (a, b) such that the four-digit number, $2a6b$, is a multiple of 6.

(a) 5   (b) 12   (c) 16   (d) 17   (e) 20

Answer: (c)

45. Two points P and Q lie on a sphere with a radius of 10. The length of the line segment joining P and Q is 10. What is the shortest distance between the segment $\overline{PQ}$ and the center of the sphere?

(a) 10   (b) 5   (c) $5\sqrt{7}$ (d) $10\sqrt{3}$ (e) $5\sqrt{3}$

Answer: (e)

46. The population in town A declined at a constant rate from 20,000 in the year 1990 to 19,120 in the year 1998. The population in town B increased at a constant rate from 12,000 in the year 1994 to 12,640 in the year 1998. If the rates of change of the population in the two towns remain the same, in approximately what year will the populations in the two towns be equal?

(a) 2020   (b) 2022   (c) 2023   (d) 2024   (e) 2025

Answer: (b)

47. Suppose that $f(x) = ax + b$, where a and b are real numbers. Given that $f(f(f(x))) = 27x + 39$, what is the value of $a + b$?

(a) 5   (b) 6   (c) 8   (d) 16   (e) 21

Answer: (b)

48. Suppose that a flight from Atlanta to Los Angeles takes 5 hours, while a flight from Los Angeles to Atlanta takes 3 hours. The difference in time is due to a wind blowing from west to east. How long, in minutes, would it take to fly between the two cities if there were no wind?

(a) 200   (b) 210   (c) 215   (d) 225   (e) 230

Answer: (d)

49. The width of a rectangle is 35% of the length. The perimeter of the rectangle is 108 cm. Find the width of the rectangle.

(a) 7 cm   (b) 14 cm   (c) 20 cm   (d) 40 cm   (e) 44 cm

Answer: (b)

50. A water tank had some water, and 3 gallons of water was added to the tank each day starting from March 1$^{\text{st}}$. If there are 110 gallons of water in the tank at the end of April 5$^{\text{th}}$, how many gallons were in the tank before March 1$^{\text{st}}$?

(a) 2   (b) 5   (c) 8   (d) 11   (e) 14

Answer: (a)

51. There are 3 red balls and 7 blue balls in Bowl A, and 8 red balls and 2 blue balls in Bowl B. Mandy rolls a dice and if a prime number is up, Bowl A is selected; otherwise Bowl B is selected. Then Katie draws a ball randomly from the selected bowl. Find the probability of Bowl A being selected given that a red ball is drawn.

Answer: $\frac{3}{11}$

52. Let $f(x) + f(1-x) = 6$ for all x. Find $f(\frac{1}{100}) + f(\frac{2}{100}) + \cdots + f(\frac{99}{100})$.

Answer: 297

53. A circle is inscribed in the triangle ABC whose side lengths are 8 cm, 10 cm, and 14 cm. Find the radius of the circle in cm.

Answer: $\sqrt{6}$