NMC Sample Problems: Grade 9

1. One root of the cubic polynomial $x^3 + 2x^2 - 4x + 1$ is 1. What is the sum of the other two roots of this polynomial?

(a) 1   (b) -1 (c) 3   (d) -3 (e) None of these

Answer: (d)

2. A pouch contains two red balls, three blue balls and one green ball. What is the probability of drawing a red ball and the green ball if two balls are drawn randomly?

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{2}{6}$ (d) $\frac{2}{15}$ (e) None of these

Answer: (d)

3. A line of slope 2 passes through the points $(2, 4)$ and $(3, b)$. Find the value b.

(a) 8   (b) 6   (c) 4   (d) 2   (e) None of these

Answer: (b)

4. Find the solution of the system of equations $\begin{cases} 3x + 6y = -3 \\ 2x + y = -8 \end{cases}$.

(a) $(5, -2)$ (b) $(-5, 2)$ (c) $(2, -5)$ (d) $(-2, -5)$ (e) None of these

Answer: (b)

5. If $f(x) = 3x^2$, what is $f(f(x))$?

(a) $3x^4$ (b) $9x^2$ (c) $9x^4$ (d) $27x^4$ (e) None of these

Answer: (d)

6. There is an isosceles triangle which is also a right triangle. Its area is 50 square inches. Find the length of the base of the triangle.

(a) $2\sqrt{5}$ inches   (b) 5 inches   (c) $5\sqrt{2}$ inches   (d) $4\sqrt{10}$ inches   (e) None of these

Answer: (e)

7. Suppose that a, b, c, d are positive integers satisfying $\frac{3a}{b} = \frac{2c}{d}$. Which of the following is not necessarily true?

(a) $a : b = 2c : 3d$ (b) $3a : 2b = c : d$ (c) $3ad = 2bc$ (d) $\frac{3a+2b}{b} = \frac{2c+2d}{d}$

(e) None of the above

Answer: (e)

8. What is the twenty-first term in the arithmetic sequence 2, 9, 16, 23, $\cdots$?

(a) 142   (b) 149   (c) 156   (d) 163   (e) None of these

Answer: (a)

9. What is the sum of the first 21 terms of the arithmetic sequence 2, 9, 16, 23, $\cdots$?

(a) 1511   (b) 1660   (c) 1816   (d) 1979   (e) None of these

Answer: (e)

10. Solve the equation $|x + 4| = 3 - 2x$ for x.

(a) $7, -\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 7   (d) $7, \frac{1}{3}$ (e) None of these

Answer: (b)

11. From a group of twenty people, how many ways can the officers: President, Vice-president, Secretary, and Treasurer be chosen?

(a) 4   (b) 24   (c) $_{20}C_4$ (d) $_{20}P_4$ (e) None of these

Answer: (d)

12. (Refer to Problem #11) The President wants two officers and two non-officers to join him on a walk. How many ways are there to choose his four fellow walkers?

(a) 360   (b) 720   (c) 1440   (d) 2880   (e) None of these

Answer: (a)

13. A ball is thrown up into the air from a patio in the two-story building which is at 30 feet high. The ball will be h feet above the ground at t seconds, and $h = 30 + 128t - 16t^2$. Find the maximum height in feet which the ball can reach.

(a) 286   (b) 316   (c) 226   (d) 512   (e) None of these

Answer: (a)

14. A square is inscribed in a circle of radius r. What fraction of the area of the circle lies outside the square?

(a) $\frac{r}{\pi}$ (b) $\frac{\pi - 2}{\pi}$ (c) $\frac{\pi r}{\pi - \sqrt{2}}$ (d) $\frac{\pi r}{2}$ (e) None of these

Answer: (b)

15. A newly discovered element Yourelementium, has a half-life of 8 months. How much of a 20 g sample of this new element will remain after 24 months? (Note: The half-life of a chemical substance is the time required for half of a given substance to disintegrate.)

(a) 0 g   (b) 5 g   (c) 2.5 g   (d) 1.25 g   (e) None of these

Answer: (c)

16. If you convert 315_9 to Base Ten, what number do you have?

    (a) 72   (b) 257   (c) 380   (d) 729   (e) None of these

Answer: (b)

17. A right triangle has two legs of length 5 and 12. What is the length of the altitude to the hypothenuse?

    (a) $\frac{12}{5}$ (b) $\frac{60}{13}$ (c) $\frac{65}{12}$ (d) $\frac{12\sqrt{2}}{5}$ (e) None of these

Answer: (b)

18. How many diagonals does an nonagon have?

    (a) 44   (b) 36   (c) 27   (d) 14   (e) None of these

Answer: (c)

19. Farmer Bob has 400 ft of fencing to enclose a rectangular field. What is the maximum possible area (in ft^2) that he can enclose?

    (a) 10000   (b) 16000   (c) 20000   (d) 40000   (e) None of these

Answer: (a)

20. Let the three roots of the equation $x^3 - 3x^2 - 4x - 7 = 0$ be a, b and c. What is $a^2b^2c + a^2bc^2 + ab^2c^2$?

    (a) 12   (b) -12 (c) -21 (d) -28 (e) None of these

Answer: (d)

21. If Alice's *i*pad crashes, she will be angry. Which of the following statement is also true?

    (a) If Alice is angry, then her *i*pad has crashed.

    (b) If Alice is not angry, then her *i*pad has not crashed.

    (c) If Alice's *i*pad does not crash, then she will be angry.

    (d) Either Alice is not angry or her *i*pad has crashed.

    (e) None of the above

Answer: (b)

22. What is the sum of the coefficients of the expression $(3a + 2b + c)^4$ when expanded and simplified?

    (a) 6   (b) 36   (c) 625   (d) 1296   (e) None of these

Answer: (d)

23. How many ways can I arrange the six letters of the word LIGHTS such that no vowel occupies an odd-numbered position?

    (a) 60   (b) 120   (c) 360   (d) 720   (e) None of these

Answer: (c)

24. Alice, Bob and Chris must clean their dorm. If it takes Alice 1 hour to clean it alone, Bob 2 hours to clean it alone, and Chris 1.5 hours to clean it alone, how many hours does it take the three of them working together to finish the task?

(a) $\frac{5}{12}$ hours (b) 1.5 hours (c) 0.6 hours (d) 0.5 hours (e) None of these

Answer: (e)

25. (Refer to Problem #24) Suppose all three begin the task at the same time. However, Alice, Bob and Chris pull the fire alarm 20 minutes after they begin. What fraction of the task remains?

(a) $\frac{6}{13}$ (b) $\frac{7}{13}$ (c) $\frac{7}{18}$ (d) $\frac{5}{18}$ (e) None of these

Answer: (d)

26. Let $a = \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots}}}$. Then a is a real number. Find a.

(a) $\sqrt{2} + 1$ (b) $\sqrt{2} - 1$ (c) $-1 - \sqrt{2}$ (d) $\frac{2}{5}$ (e) None of these

Answer: (b)

27. Alice has two bags of marbles: Bag A contains 8 red and 3 blue marbles, and Bag B contains 5 red and 7 blue marbles. She randomly selects a bag and then draws one marble at random. It is blue. What is the probability that it came from Bag B?

(a) $\frac{7}{12}$ (b) $\frac{7}{10}$ (c) $\frac{77}{113}$

(d) Not enough information (e) None of these

Answer: (c)

28. How many positive integral divisors does the number 216 have?

(a) 16 (b) 17 (c) 18 (d) 19 (e) None of these

Answer: (a)

29. Compute $\phi(2015)$, where $\phi(n)$ is the number of positive integers less than and relatively prime to n.

(a) 3 (b) 1440 (c) 1608 (d) 2014 (e) None of these

Answer: (b)

30. How many ordered triples of positive integers (x, y, z) satisfy $x + y + z \le 7$?

(a) 11 (b) 18 (c) 24 (d) 44 (e) None of these

Answer: (e)

31. What is the coefficient of xy^2z in the expansion of $(x + y + z)^4$?

(a) 3   (b) 4   (c) 6   (d) 12   (e) None of these

Answer: (d)

32. What is the probability of rolling a sum of 11 with three standard six-sided dice?

(a) $\frac{11}{216}$ (b) $\frac{1}{8}$ (c) $\frac{25}{216}$ (d) $\frac{7}{72}$ (e) None of these

Answer: (b)

33. A square with vertices $(0,0)$, $(2,0)$, $(0,2)$ and $(2,2)$ is rotated around the y-axis. Compute the volume of the resulting figure.

(a) 4π (b) 8π (c) 12π (d) 16π (e) None of these

Answer: (b)

34. What is the sum of all prime factors of 2015?

(a) 49   (b) 50   (c) 2064   (d) 2065   (e) None of these

Answer: (a)

35. Solve the inequality $|4 - 5x| \leq 8$ for x.

(a) $-\frac{4}{5} \leq x \leq \frac{12}{5}$ (b) $x < -\frac{4}{5}$ or $x > \frac{12}{5}$ (c) $-\frac{12}{5} \leq x \leq \frac{4}{5}$

(d) $x \geq \frac{4}{5}$ or $x \leq -\frac{12}{5}$ (e) None of these

Answer: (a)

36. What is the set of points equidistant from $(3, 3)$ and $(6, 0)$.

(a) circle   (b) line   (c) parabola   (d) hyperbola   (e) None of these

Answer: (b)

37. The area of triangle ABC is 18. If the coordinates of A and B are $(-3, 2)$ and $(3, 2)$ respectively, what is the locus of point C?

(a) $y = 3x + 3$ (b) $y = 8$ or $y = -4$ (c) $y = 3$ or $y = -3$

(d) $x = 6$ or $x = -6$ (e) None of these

Answer: (b)

38. In New-Kid's-Town, required bicycle licenses consist of two letters followed by three digits. (For example, LV123, AA000, MP906, ZZ999, and so on.) How many distinct license numbers are available?

(a) $676,000$ (b) $628,600$ (c) $628,340$ (d) $300,600$ (e) None of these

Answer: (a)

39. Given that $x - 1$ and $x + 1$ are reciprocal of each other, what is x^2?

(a) 2   (b) $\sqrt{2}$ (c) 4   (d) 1   (e) None of these

Answer: (a)

40. The length of the shortest side of a right triangle is 11. If the lengths of all three sides are integers, what is the perimeter of the triangle?

(a) 241   (b) 198   (c) 132   (d) 121   (e) None of these

Answer: (c)

41. Consider point $P = (2, 3)$ on the Cartesian plane. If you first reflect P about the line $y = x$ and then rotate it about the origin by $90°$ counterclockwise, what are the coordinates of the new position of P?

(a) $(-2, 3)$ (b) $(-2, -3)$ (c) $(2, -3)$ (d) $(3, 2)$ (e) None of these

Answer: (a)

42. For what real values of x does the expression below represent a real number?
$$\sqrt{\frac{x + 4}{2 - x}}$$

(a) $x < 2$ (b) $-4 \le x < 2$ (c) $-3 \le x \le 4$ (d) $-3 \le x$ (e) None of these

Answer: (b)

43. What is the value of $\dfrac{a^2 - b^2 - c^2}{bc}$ if $a = b + c$?

(a) 1   (b) 2   (c) 3   (d) 4   (e) None of these

Answer: (b)

44. Solve the inequality
$$|3x - 4| < x + 5.$$

(a) $0 < x < 4$ (b) $-\frac{1}{8} < x < 94$ (c) $-\frac{1}{8} < x < 3$ (d) $-\frac{1}{4} < x < \frac{9}{2}$ (e) None of these

Answer: (d)

45. Find the area of parallelogram $ACDB$ with vertices
$$A = (0, 0),\ B = (3, 1),\ C = (4, 4),\ D = (7, 5).$$

(a) 11   (b) 10   (c) 9   (d) 8   (e) None of these

Answer: (d)

46.   Consider functions f whose domain is given by $\{1, 2, 3\}$ and whose values lie in the set of integers $\{0, 1, 2, 3, 4\}$. How many different functions f can be formed?

   (a) 60   (b) 100   (c) 125   (d) 243   (e) None of these

   Answer: (c)

47.   A chemist mixes distilled water with a 90% solution of sulfuric acid to produce a 50% solution. If 5 liters of distilled water is used, how much 50% solution is produced? (Answer in liters using decimals.)

   (a) 10.75   (b) 11.25   (c) 12.5   (d) 13.75   (e) None of these

   Answer: (b)

48.   Evaluate the sum of the first 21 values $\frac{1}{\sqrt{3n+1}+\sqrt{3n-2}}$ starting from $n = 1$ (up to $n = 21$): that is the value

$$\frac{1}{\sqrt{4}+\sqrt{1}} + \frac{1}{\sqrt{7}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{64}+\sqrt{61}}.$$

   (a) $\frac{7}{3}$ (b) $\frac{8}{3}$ (c) 3   (d) $\frac{11}{3}$ (e) None of these

   Answer: (a)

49.   What is the sum of the measures of the interior angles of an hexagon? (Answer in degrees.)

   (a) 360   (b) 480   (c) 600   (d) 720   (e) None of these

   Answer: (d)

50.   A 6-faced die is tossed twice. What is the probability that the same numbers are shown in both toss?

   (a) $\frac{1}{36}$ (b) $\frac{1}{18}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$ (e) None of these

   Answer: (c)

51.   How many ordered pairs (x, y) of positive integers x and y satisfy the equation $3x + 5y = 80$?

   Answer: 5 (pairs)

52.   What is the area of a regular hexagon inscribed in a circle with radius 1? (Answer in fraction.)

   Answer: $\frac{3\sqrt{3}}{2}$

53.   If a fair coin is tossed 3 times, what is the probability of getting at least 2 heads ("at least 2 heads" means 2 or more heads)?

   Answer: $\frac{1}{2}$