NMC Sample Problems: Grade 10

1. Burger Queen advertises, "Our French fries is 25% larger than MacTiger's fries at a price 25% less than MacTiger's". For the same size, by how much, in percentage, are Burger Queen's fries cheaper than MacTiger's?

(a) 20% (b) 25% (c) 30% (d) 40% (e) 45%

Answer: (d)

2. Simplify $\sqrt[3]{a^2\sqrt[4]{a\sqrt{a}}}$, where a is a positive real number.

(a) $a^{19/24}$ (b) $a^{1/2}$ (c) $a^{-1/2}$ (d) $a^{5/2}$ (e) None of these

Answer: (a)

3. Let $f(x) = \dfrac{2x-3}{5x+4}$. Find the value that $f(x)$ cannot assume.

(a) 1 (b) $\frac{4}{5}$ (c) $-\frac{4}{5}$ (d) $\frac{2}{5}$ (e) $-\frac{3}{4}$

Answer: (d)

4. How many factors does 280 have?

(a) 10 (b) 16 (c) 28 (d) 100 (e) None of these

Answer: (b)

5. What is the sum of the positive factors of 280?

(a) 150 (b) 280 (c) 560 (d) 720 (e) None of these

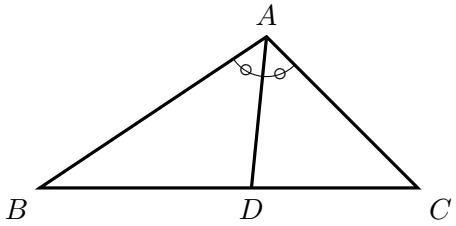
Answer: (d)

6. What is the least positive integer with exactly 12 positive factors?

(a) 12 (b) 24 (c) 60 (d) 72 (e) None of these

Answer: (c)

7. Find CD in the figure below if $\overline{AB} = 6$, $\overline{BD} = 4.5$, $\overline{AC} = 3$ and $\overline{AD}$ bisects $\angle BAC$.



(a) 1.5 (b) 2.0 (c) 2.25 (d) 3.0 (e) None of these

Answer: (c)

8. If $x < 0$ and $0 < y < 1$, which of the following is true?

(a) $x < xy^2 < xy$ (b) $x < xy < xy^2$ (c) $xy^2 < x < xy$

(d) $xy^2 < xy < x$ (e) None of these

Answer: (b)

9. The Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, $\cdots$ starts with two 1s, and each term afterwards is the sum of its two predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

(a) 2 (b) 5 (c) 6 (d) 8 (e) 9

Answer: (c)

10. Solve $|3x + 6| = |2x - 1|$.

(a) $x = 2$ (b) $x = -7$ (c) $x = -1$ (d) $x = -7, -1$ (e) $x = 0$

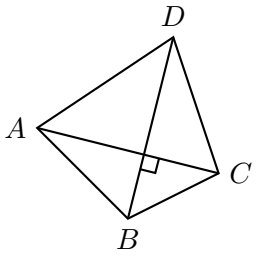
Answer: (d)

11. What are the x-intercepts of $|x| + |y| = 3$?

(a) $x = 0, 3$ (b) $x = 0, -3$ (c) $x = -1, 5$ (d) $x = -3, 3$ (e) None of these

Answer: (d)

12. In the figure below $\overline{AC}$ and $\overline{BD}$ are perpendicular, $\overline{AD} = 6$, and $\overline{BC} = 4$. Find $\overline{AB}^2 + \overline{CD}^2$.



(a) 12 (b) 24 (c) 52 (d) 144

(e) Cannot be determined

Answer: (c)

13. If $27^{2t} = 64$, then what is the value of 9^{2t-2}?

(a) 5/12 (b) $\frac{4}{5}$ (c) $\frac{5}{64}$ (d) $\frac{16}{81}$ (e) None of these

Answer: (d)

14. For $f(x) = \dfrac{x-3}{x+1}$, define $f_1(x) = f(x)$, $f_2(x) = f(f_1(x))$, $f_3(x) = f(f_2(x))$, and so on. Evaluate $f_{2009}(f_{2010}(3))$.

(a) -2 (b) -3 (c) 3 (d) 8 (e) 9

Answer: (b)

15. What is the sum of $-148 - 130 - \cdots + 50 + 68 + 86$?

(a) -2 (b) -132 (c) -298 (d) -434 (e) None of these

Answer: (d)

16. If x, y, and z are positive integers such that $x^2 + y^2 + z^2 = 147$. What is the greatest value of $x + y + z$?

(a) 12 (b) 13 (c) 20 (d) 21 (e) 29

Answer: (d)

17. Solve the equation $2\sin^2 x - \sin x - 1 = 0$, $x \in [0, 2\pi]$.

(a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (d) $\frac{\pi}{3}, \frac{2\pi}{3}$ (e) None of these

Answer: (c)

18. When the mean, median, and mode of the list

$$10, 2, 5, 2, 4, 2, x$$

are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real value of x?

(a) 15 (b) 20 (c) 30 (d) 48 (e) None of these

Answer: (b)

19. What is the sum of all numbers x such that $(3x^2 + 9x - 2012)^{x^3 - 2012x^2 - 10x + 1} = 1$?

(a) 0 (b) 1 (c) 2000 (d) 2009 (e) 2020

Answer: (d)

20. Suppose that α and β are nonzero real solutions of the equation $2x^2 - x + 1 = 0$. Find the sum $\frac{1}{\alpha} + \frac{1}{\beta}$.

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

Answer: (d)

21. For an equilateral triangle with a side length 9, find the area of the circumscribing circle.

(a) 10π (b) 18π (c) 27π (d) 81π (e) None of these

Answer: (c)

22. Simplify the following expression

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots+\frac{1}{\sqrt{24}+\sqrt{25}}$$

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

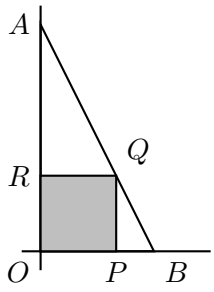
Answer: (d)

23. Arrange the numbers $a = 6^{50}$, $b = 3^{75}$, and $c = 2^{125}$ from smallest to greatest.

(a) $a < b < c$ (b) $a < c < b$ (c) $b < a < c$ (d) $b < c < a$ (e) None of these

Answer: (d)

24. Let a point Q be on the line segment connecting $A(0, 12)$ and $B(4, 0)$. Find the maximum area of the rectangle $\square\, OPQR$.



(a) 12 (b) 24 (c) 30 (d) 40 (e) None of these

Answer: (a)

25. Find the constant term in the expansion of $\left(3x^2+\frac{1}{x}\right)^6$.

(a) 112 (b) 124 (c) 135 (d) 240 (e) None of these

Answer: (c)

26. If $xyz = 8, x, y$, and z are positive, find the minimum value of $x + y + z$.

(a) 1 (b) 2 (c) 4 (d) 6 (e) 8

Answer: (d)

27. Find $\tan(\arccos x)$ in terms of x.

(a) $\frac{1}{x}$ (b) $\frac{\sqrt{1-x^2}}{x}$ (c) $\frac{1-x}{x}$ (d) x (e) None of these

Answer: (b)

28. Find the period of $f(x) = -2\sin(3x - \frac{\pi}{2})$.

(a) π (b) 2π (c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{2}$ (e) None of these

Answer: (c)

29. Find the phase shift of $f(x) = 3\cos(3x + \frac{\pi}{2})$.

(a) π (b) 2π (c) $\frac{\pi}{6}$ (d) $-\frac{\pi}{6}$ (e) None of these

Answer: (d)

30. How many zeros does $\dfrac{50!}{2^9 5^5}$ end in?

(a) 1 (b) 3 (c) 5 (d) 7 (e) 8

Answer: (d)

31. What is the exact value of $|\pi - \frac{25}{8}| + |\pi - \frac{16}{5}|$? Write your answer as a common fraction.

(a) $\dfrac{21}{8}$ (b) $\dfrac{9}{40}$ (c) $\dfrac{3}{40}$ (d) $\dfrac{17}{40}$ (e) None of these

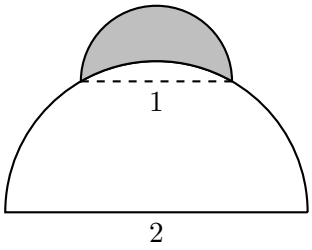
Answer: (c)

32. Suppose that $3 - 2\sqrt{2}$ solves $x^2 - 6x + a = 0$. Find the value of a.

(a) 1 (b) 3 (c) 5 (d) 7 (e) 8

Answer: (a)

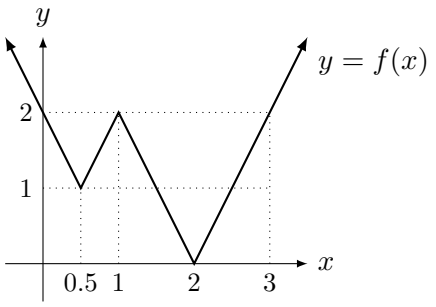
33. Find the shaded area.



(a) $\frac{\sqrt{3}}{4} - \frac{1}{24}\pi$ (b) π (c) 5 (d) 10 (e) $\frac{\pi}{6}$

Answer: (a)

34. For a graph of $y = f(x)$ shown below, find the number of real numbers x satisfying $f(f(x)) = 1$.



(a) 1 (b) 3 (c) 5 (d) 7 (e) 8

Answer: (e)

35. Consider the circle $(x-3)^2 + (y-4)^2 = 8$. Find an equation of the line tangent to the circle at the point $(1, 2)$.

(a) $y = x + 3$ (b) $y = -x + 3$ (c) $y = 2x + 6$ (d) $y = x + 6$ (e) $y = 3x + 3$

Answer: (b)

36. If a function $f(x)$ satisfies $2f(x) + 3f(1-x) = x^2$ for all real numbers x, what is $f(0)$?

(a) 1 (b) $\frac{3}{5}$ (c) $-\frac{4}{5}$ (d) $\frac{2}{5}$ (e) $-\frac{3}{4}$

Answer: (b)

37. Jennifer is visiting her mother living in a town 60 miles from her house. She drove 40 mph to her mother's house and 30 mph on the way back. What is her average speed for the trip?

(a) 36 mph (b) 34 (c) $\frac{240}{7}$ mph (d) $\frac{250}{7}$ (e) $\frac{246}{7}$

Answer: (c)

38. Jim parked his car in a parking lot at a randomly chosen time between 2:30 PM and 4:00 PM. Exactly half an hour later he drove his car out of the parking lot. What is the probability that he left the parking lot after 4:00 PM?

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$ (e) $\frac{2}{5}$

Answer: (a)

39. Two numbers, x and y are selected at random from the interval $(0, 3)$. What is the probability that $y \geq x + 1$?

(a) 2/7 (b) 1/7 (c) 1/3 (d) 4/9 (e) 2/9

Answer: (e)

40. How many non-negative integer solutions are there to the equation $x + y + z + w = 10$?

(a) 280 (b) 300 (c) 286 (d) 264 (e) 272

Answer: (c)

41. How many ways can I select 10 cans of soda from a cooler containing large quantities of Coke, Pepsi, Dr. Pepper, and Sprite?

(a) 300 (b) 264 (c) 240 (d) 286 (e) 306

Answer: (d)

42. A die is tossed 5 times. Compute the probability that a 3 comes up exactly 3 times (out of 5 trials).

(a) $\frac{25}{972}$ (b) $\frac{125}{3888}$ (c) $\frac{100}{3751}$ (d) $\frac{50}{1877}$ (e) $\frac{100}{3957}$

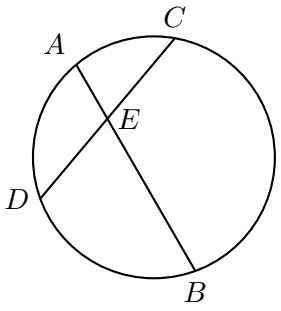
Answer: (b)

43. Square $ABCD$ has sides of length 10 units. Isosceles triangle GDC has base DC, and the area common to triangle GDC and square $ABCD$ is 80 square units. Find the length of the altitude to DC in $\triangle GDC$.

(a) 25 (b) 20 (c) 24 (d) 28 (e) 30

Answer: (a)

44. In the circle below, $\overline{AB}$ and $\overline{CD}$ are chords intersecting at E (Not drawn to scale). If $\overline{AE} = 5$, $\overline{BE} = 14$, and $\overline{CE} = 7$, what is the length of $\overline{DE}$?



(a) 8 (b) 17/2 (c) 19/2 (d) 10 (e) 45/4

Answer: (d)

45. John forgot to write down a very important phone number. All he remembers is that it started with 866 and that the next set of 4 digits involved 1, 3 and 9 with one of these numbers appearing twice. If he guesses a phone number, what is the probability that he gets the number correct?

(a) 1/24 (b) 1/36 (c) 2/75 (d) 3/64 (e) 3/56

Answer: (b)

46. For a real number x find the sum of the maximum and minimum of

$$y = \frac{(x^2 - 2x - 3)}{(2x^2 + 2x + 1)}.$$

(a) 2 (b) 5 (c) -5 (d) 0 (e) -3

Answer: (e)

47. What is the sum of $2^0 + 2^1 + 2^2 + ... + 2^{19}$?

(a) 1013466 (b) 1034676 (c) 1034677 (d) 1048575 (e) 1062442

Answer: (d)

48. 10 monkeys find an incredibly large ship filled with bananas stranded on the shore. The k^{th} monkey takes $\frac{k}{10}$ of the bananas that remain on the ship. If every monkey took a positive whole number of bananas, what is the smallest number of bananas that the 10^{th} monkey will take?

(a) 543 (b) 567 (c) 588 (d) 536 (e) 563

Answer: (b)

49. Jisu and Junho leave their apartment building at the same time. The building is located on a circular walkway. Jisu and Junho walk, in opposite directions, around the circle. Jisu takes 2 minutes to walk around the circle, and Junho takes 2.2 minutes to walk around the circle. At some random time between 9 and 11 minutes after they leave, a photographer comes and, standing inside the circle, takes a picture of $\frac{1}{4}$ of the circle, with the apartment building directly in the center of the picture. What is the probability that neither Jisu nor Junho are in the picture?

(a) $\frac{11}{20}$ (b) $\frac{25}{40}$ (c) $\frac{13}{20}$ (d) $\frac{29}{40}$ (e) $\frac{3}{4}$

Answer: (c)

50. What is the remainder when 7^{12} is divided by 12?

(a) 1 (b) 2 (c) 3 (d) 5 (e) 6

Answer: (a)

51. What is the hundreds digit of 1989^{1989}?

Answer: 5

52. How many integers between 1 and 1500 (inclusive) are divisible by 3 or 5?

Answer: 700

53. How many pairs of positive integers (a, b) exist that satisfy the equation $a^2 + b^2 = a^3$?

Answer: Infinitely many