NMC Sample Problems: Grade 11

1. Which one of the following functions has different domain?

(a) $\frac{x+1}{x-1}$ (b) $\frac{(x+1)\sqrt{x^2+1}}{(x-1)\sqrt{x^2+1}}$ (c) $\frac{(x+1)(x+2)}{(x-1)(x+2)}$ (d) $\frac{(x+1)(x^2+2)}{(x-1)(x^2+2)}$ (e) $\frac{(x+1)(x^3+1)}{x-1}$

Answer: (c)

2. Find the equation of the circle centered at $(1,-2)$ passing through the point $(-2,2)$.

(a) $(x-1)^2+(y+2)^2=5$ (b) $(x-1)^2+(y+2)^2=25$

(c) $(x+1)^2+(y-2)^2=5$ (d) $(x+1)^2+(y-2)^2=25$

(e) $(x-2)^2+(y+2)^2=5$

Answer: (b)

3. What is the solution set of the following equation: $\ln(x)+\ln(x+2)=\ln(x+6)$?

(a) $\{1\}$ (b) $\{2\}$ (c) $\{2,-3\}$ (d) $\{2,3\}$ (e) $\{1,2\}$

Answer: (b)

4. Find the polar coordinates for the point with rectangular coordinates $(0,-2)$.

(a) $(2,0)$ (b) $(2,\frac{\pi}{2})$ (c) $(2,\pi)$ (d) $(2,\frac{3\pi}{2})$ (e) $(2,2\pi)$

Answer: (d)

5. When $\sin\theta=\frac{1}{7}$ and $\tan\theta>0$, find $\sin(\theta+\frac{\pi}{2})$.

(a) $-\frac{1}{7}$ (b) $\frac{1}{7}$ (c) $-\frac{4\sqrt{3}}{7}$ (d) $\frac{4\sqrt{3}}{7}$ (e) $\frac{6}{7}$

Answer: (d)

6. If $\sin x-\cos x=\frac{3}{5}$, then what is the value of $\sin^3 x-\cos^3 x$?

(a) $\frac{4}{5}$ (b) $\frac{27}{125}$ (c) $\frac{33}{125}$ (d) $\frac{66}{125}$ (e) $\frac{99}{125}$

Answer: (e)

7. There are 3 red balls, 2 blue balls, and 3 white balls in a bag. If James takes out three balls at once, then what is the probability that he takes out at least one red ball?

(a) 1 (b) $\frac{27}{28}$ (c) $\frac{25}{28}$ (d) $\frac{23}{28}$ (e) $\frac{21}{28}$

Answer: (d)

8. Let x and y be real numbers satisfying the following two conditions:

$$x + y = 1 \quad \text{and} \quad (x^2 + y^2)(x^3 + y^3) = 35$$

Find $x^2 + y^2$.

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

Answer: (c)

9. Let a_7, a_6, $\cdots$, a_1 and a_0 be the coefficients of the expansion of the expression $(2x - 3)^7$, that is

$$(2x - 3)^7 = a_7x^7 + a_6x^6 + \cdots + a_1x + a_0$$

Find the sum $a_7 + a_6 + \cdots + a_1 + a_0$.

(a) -1 (b) 0 (c) 1 (d) 7 (e) 2^7

Answer: (a)

10. Find the sum $3^{-1} + 3^{-2} + \cdots + 3^{-8}$. (Hint: $3^8 = 6561$)

(a) $\frac{3270}{6561}$ (b) $\frac{3280}{6561}$ (c) $\frac{3290}{6561}$ (d) $\frac{3300}{6561}$ (e) $\frac{3310}{6561}$

Answer: (b)

11. What is the remainder when $1! + 2! + 3! + \cdots + 2012!$ is divided by 7?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 0

Answer: (c)

12. How many five digit numbers contain the digit pattern "03" exactly once?

(a) 2673 (b) 2700 (c) 2691 (d) 2682 (e) 2704

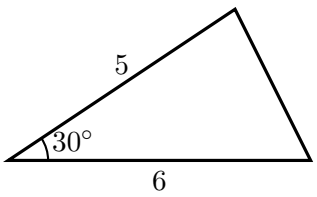
Answer: (c)

13. Let a, b and c be three roots of $x^3 - 2x^2 - 3x - 4 = 0$. Find $p(a) + p(b) + p(c)$, where $p(x) = x^5 - 2x^4 - 3x^3 - 4x^2 + x + 1$.

(a) 5 (b) 3 (c) 1 (d) -1 (e) -3

Answer: (a)

14. Find the area of the following triangle.



(a) 6 (b) 6.5 (c) 7 (d) 7.5 (e) 8

Answer: (d)

15. Suppose a, b, c and d are integers, and $x = \sqrt{2+\sqrt{3}}$ is a solution of the equation $2x^4 + ax^3 + bx^2 + cx + d = 0$. What is the sum $a + b + c + d$?

(a) 3 (b) -6 (c) 9 (d) -12 (e) 15

Answer: (b)

16. What is the remainder when $x^3 - 2x + 1$ is divided by $2x - 4$?

(a) $x^3 - 4x + 5$ (b) $x + 1$ (c) $\frac{5}{2}$ (d) 5 (e) 10

Answer: (d)

17. Find the equation of the straight line which passes through the point $(2, 3)$ and is perpendicular to $2x + 3y + 1 = 0$.

(a) $2x + 3y - 13 = 0$ (b) $2x - 3y + 5 = 0$ (c) $3x + 2y + 12 = 0$
(d) $3x - 2y = 0$ (e) $3x + 2y = 0$

Answer: (d)

18. Find the vertex point of the parabola $x = y^2 + 4y + 7$.

(a) $(3, 2)$ (b) $(3, -2)$ (c) $(-3, 2)$ (d) $(-3, -2)$ (e) $(2, 3)$

Answer: (b)

19. Find the value of the product:

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{2008^2}\right)$$

(a) $\frac{2009}{4016}$ (b) $\frac{4016}{2009}$ (c) $\frac{2009}{8032}$ (d) $\frac{8032}{2009}$ (e) $\frac{12048}{2009}$

Answer: (a)

20. How many real number solutions of the equation $e^x + x^2 - 2 = 0$?

(a) 0 (none) (b) 1 (one solution) (c) 2 (two solutions)
(d) 3 (three solutions) (e) 6 (six solutions)

Answer: (c)

21. A ball is thrown vertically upward and its height after t seconds is $s = 120 + 64t - 16t^2$ feet. Find the maximum height reached by the ball.

(a) 184 feet (b) 168 feet (c) 136 feet (d) 120 feet (e) 64 feet

Answer: (a)

22. The function $f(x) = \frac{2x+1}{x-3}$ has one horizontal asymptote, say $y = a$, and one vertical asymptote, say $x = b$. Find $a + b$.

(a) -1 (b) 0 (c) 1 (d) 3 (e) 5

Answer: (e)

23. If $16^{x+1} = 3$, then which of the following is 2^{4x+2}?

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) 3 (d) 6 (e) 12

Answer: (a)

24. If $\csc(x) = 3$ and $90^\circ < x < 180^\circ$, then what is $\cos(x)$?

(a) $-\frac{7}{9}$ (b) $-\frac{2\sqrt{2}}{3}$ (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$ (e) $\frac{2\sqrt{2}}{3}$

Answer: (b)

25. If $\sin x + \cos x = \frac{1}{3}$, then what is the value of $\sin^3 x + \cos^3 x$?

(a) $\frac{2\sqrt{2}}{3}$ (b) $-\frac{2\sqrt{2}}{3}$ (c) $\frac{7}{9}$ (d) $\frac{13}{27}$ (e) $-\frac{13}{27}$

Answer: (d)

26. Evaluate $\log_{1/27} 9$?

(a) -2 (b) $-\frac{4}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{2}{3}$ (e) $\frac{4}{3}$

Answer: (c)

27. Find the remainder when $1 + 3 + 5 + \cdots + 2007 + 2009$ is divided by 2009.

(a) 1507 (b) 1307 (c) 1005 (d) 503 (e) 0

Answer: (a)

28. Find the sum of the maximum and minimum values of the expression $-2\sin(2x + \pi) + 5$.

(a) 3 (b) 7 (c) 10 (d) 13 (e) 15

Answer: (c)

29. Evaluate the following: $\left(\dfrac{\sqrt{2}}{1+i}\right)^{2009}$

(a) 0 (b) 1 (c) $\frac{\sqrt{2}(1-i)}{2}$ (d) $\frac{\sqrt{2}(1+i)}{2}$ (e) $-\frac{\sqrt{2}(1+i)}{2}$

Answer: (c)

30. Find the maximum value of $\sin x - 2\cos x + 3$.

(a) 3 (b) 2 (c) $3 + \sqrt{5}$ (d) 6 (e) $3 - \sqrt{5}$

Answer: (c)

31. In the expansion of $(x^2 - 2y)^{10}$, what is the coefficient of $x^{14}y^3$?

(a) -960 (b) 480 (c) -240 (d) 1 (e) 0

Answer: (a)

32. Find the solution set of the equation $\left(e^{x+1}\right)^{x+2} = 1$.

(a) $\{1, 2\}$ (b) $\{-1, -2\}$ (c) $\{\frac{3}{2}\}$ (d) $\{-\frac{3}{2}\}$ (e) $\{\}$ (No solution)

Answer: (b)

33. A committee composed of Alice, Mark, Ben, Connie and Francisco is about to select three representatives randomly. What is the probability that Connie is excluded from the selection?

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$ (e) 0

Answer: (c)

34. Find the sum of all integers between 1 to 100 (inclusive) that are not multiples of 3.

(a) 3367 (b) 3368 (c) 3369 (d) 3370 (e) 3375

Answer: (a)

35. Evaluate the following:

$$\log_2\left(\frac{1}{2}\right) + \log_2\left(\frac{2}{3}\right) + \log_2\left(\frac{3}{4}\right) + \cdots + \log_2\left(\frac{1023}{1024}\right)$$

(a) -10 (b) -5 (c) 0 (d) 5 (e) 10

Answer: (a)

36. How many one-to-one functions are there from the set $X = \{a, b, c\}$ to the set $Y = \{1, 2, 3, 4\}$?

(a) 9 (b) 16 (c) 24 (d) 27 (e) 64

Answer: (c)

37. Let $z = 2 + bi$, where b is a positive real number and $i^2 = -1$. If the imaginary part of z^2 and z^3 are equal, then what is b?

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$ (e) 4

Answer: (d)

38. What is the average of the numbers in the set $\{2, 6, 10, 14, \cdots, 38\}$?

(a) 22 (b) 21 (c) 20 (d) 19 (e) 18

Answer: (c)

39. The rational function $f(x) = \frac{ax+2}{x+b}$ has the inverse function $f^{-1}(x) = \frac{ax+2}{x+b}$. Find $a + b$.

(a) -2 (b) $-\frac{1}{2}$ (c) 0 (d) $\frac{1}{2}$ (e) 2

Answer: (c)

40. What is the area of a triangle when its three sides are 3, 6, and 7?

(a) 4 (b) 5 (c) $4\sqrt{3}$ (d) $4\sqrt{5}$ (e) $5\sqrt{3}$

Answer: (d)

41. An employee of a computer store is paid a base salary \$1,500 a month plus a 2% commission on all sales over \$4,000 during the month. How much must the employee sell in a month to earn a total of \$2,000 for the month?

(a) \$25,000 (b) \$26,000 (c) \$27,000 (d) \$28,000 (e) \$29,000

Answer: (e)

42. A speedboat takes 1 hour longer to go 16 miles up a river than to return. If the boat cruises at 12 miles per hour in still water, what is the rate of the current?

(a) 2 miles/hour (b) 4 miles/hour (c) 5 miles/hour (d) 6 miles/hour (e) 8 miles/hour

Answer: (b)

43. The domain of the function

$$f(x) = \log_{2010}(\log_{2009}(\log_{2008}(\log_{2007} x)))$$

is $\{x|x > c\}$. What is the value of c?

(a) 0 (b) 1 (c) 2007^{2008} (d) 2009^{2010} (e) $2007^{2008^{2009}}$

Answer: (c)

44. Find the sum of all solutions of the equation $|x-1| = |2x|$.

(a) -1 (b) $-\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) 0 (e) $\frac{1}{3}$

Answer: (b)

45. Which one of the following is the greatest?

(a) 7^{10} (b) 6^{20} (c) 5^{30} (d) 4^{40} (e) 3^{50}

Answer: (d)

46. Suppose that m is an integer and $2+\sqrt{3}$ is a solution of the equation $x^3 - mx^2 + mx - 1 = 0$. Find m.

(a) 5 (b) 4 (c) 3 (d) 2 (e) 1

Answer: (a)

47. Harry performed the calculation: $5 \times 7 = 21$. It turns out Harry's calculation is correct in base b ($b > 7$). Find the value b.

(a) 13 (b) 14 (c) 15 (d) 16 (e) 17

Answer: (e)

48. Suppose that $f(x)$ is a polynomial with integer coefficients, having 2006 and 2022 as zeros. Which of the following could possibly be the value of $f(2011)$?

(a) 35 (b) 77 (c) 101 (d) 110 (e) 111

Answer: (d)

49. A polynomial $f(x) = x^3 + ax^2 + bx + c$ satisfies $f(1) = 1$, $f(2) = 2$ and $f(3) = 3$. What is the remainder when $f(x)$ is divided by $x + 1$?

(a) -25 (b) -1 (c) 0 (d) 1 (e) 25

Answer: (a)

50. Evaluate $\dfrac{2012^4 - 2 \cdot 2012^2 - 3 \cdot 2012 - 2}{2012^3 - 2012^2 - 2012 - 2}$.

(a) 2011 (b) 2012 (c) 2013 (d) 2014 (e) 2015

Answer: (c)

51. Find the remainder when 2012^{2012} is divided by 11. *Answer:* 1

52. If $\log_5 2 = a$, then what is $\log_4 25$ in terms of a? *Answer:* $\frac{1}{a}$

53. Let a and b be integers which satisfy $\sqrt{7 - 4\sqrt{3}} = a + b\sqrt{3}$. Find $a + b$. *Answer:* 1