Korean-American Scientists and Engineers Association

Mathematics & Science Olympiad 2021

Answer sheet – Grade X

Identification

Registration #	
Name	
School Grade	

Answers

1	ⓐ ⓑ ⓒ ⓓ ⓔ	**11**	ⓐ ⓑ ⓒ ⓓ ⓔ	**21**	
2	ⓐ ⓑ ⓒ ⓓ ⓔ	**12**	ⓐ ⓑ ⓒ ⓓ ⓔ	**22**	
3	ⓐ ⓑ ⓒ ⓓ ⓔ	**13**	ⓐ ⓑ ⓒ ⓓ ⓔ	**23**	
4	ⓐ ⓑ ⓒ ⓓ ⓔ	**14**	ⓐ ⓑ ⓒ ⓓ ⓔ	**24**	
5	ⓐ ⓑ ⓒ ⓓ ⓔ	**15**	ⓐ ⓑ ⓒ ⓓ ⓔ	**25**	
6	ⓐ ⓑ ⓒ ⓓ ⓔ	**16**	ⓐ ⓑ ⓒ ⓓ ⓔ	**26**	
7	ⓐ ⓑ ⓒ ⓓ ⓔ	**17**	ⓐ ⓑ ⓒ ⓓ ⓔ	**27**	
8	ⓐ ⓑ ⓒ ⓓ ⓔ	**18**	ⓐ ⓑ ⓒ ⓓ ⓔ	**28**	
9	ⓐ ⓑ ⓒ ⓓ ⓔ	**19**	ⓐ ⓑ ⓒ ⓓ ⓔ	**29**	
10	ⓐ ⓑ ⓒ ⓓ ⓔ	**20**	ⓐ ⓑ ⓒ ⓓ ⓔ	**30**	

►►► Official Use Only ◄◄◄

Problem [#1-#20] (3 points each)	Problem [#21-#27] (5 points each)	Problem [#28-#30] (7 points each)	Total

Korean-American Scientists and Engineers Association

Mathematics & Science Olympiad 2021

Problems – Grade X

Identification

Registration #	
Name	
School Grade	

Instructions

- Write your registration number, English name, and school grade name in the "Identification" part of **both the problem sheets and the answer sheet**.
- You have **90 minutes** to complete this exam. There are 30 questions, composed of multiple-choice problems and free-response problems. Each multiple-choice problem is worth 3 points, and each free-response problem is worth 5 points except the last three problems which are worth 7 points each. No partial credit will be given. Ties will be broken using the following factors: (A) Highest scores on the test, (B) Highest scores on free-response problems, and (C) Scores on some selected problems (#30, #29 and #28 in that order).
- **Calculator, dictionary, ruler or notes are not allowed.** Use the provided space and back of the problem sheets to solve the problems.
- Write the answers in the "Answers" section of the answer sheet! For the multiple-choice problems, select the letter (ⓐ, ⓑ, ⓒ, ⓓ, or ⓔ) representing the correct answer. For the free-response problems, write a **simplified** answer (fractions must be reduced completely, etc) unless indicated otherwise.
- Do NOT write anything in the "Official Use" part of the answer sheet.
- When you are done with the test, return both the problem sheets and the answer sheet to the proctor.

►►► Do Not Open Until Instructed! ◄◄◄

Example for 4th Grade

♦♦♦ Multiple-Choice Problems ♦♦♦

1. What is the digit in the tens place of the following: 2021 – 1976?

   (a) 2 (b) 3 (c) 4 (d) 5 (e) 6

2. Which of the following does not divide 1980?

   (a) 4 (b) 6 (c) 9 (d) 11 (e) 13

3. What is the average of the following scores: 79, 82, 84, 89, and 90?

   (a) 84.8 (b) 85 (c) 85.4 (d) 85.8 (e) 86

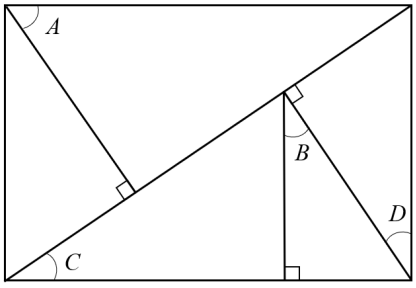
4. Elizabeth is 5 feet tall and has a shadow of 3 feet. If Elizabeth's brother is 3 feet taller than Elizabeth and he is standing next to Elizabeth, how long is his shadow?

   (a) 4.8 feet (b) 5 feet (c) 5.2 feet (d) 5.4 feet (e) 5.6 feet

5. There are 256 tennis players participating in a tennis tournament. Each player plays at most one match per day. Every player plays on the first day, only winners remain to play the next day, and so on. If the tournament started on August 30th, which of the following will be the date of the final match where only two players remain to play?

(a) Sept. 5th (b) Sept. 6th (c) Sept. 7th (d) Sept. 8th (e) Sept. 9th

6. In the following rectangle, which angle could be different from the others? (*Note: Figure not drawn to scale!*)



(a) A (b) B (c) C (d) D (e) None

♦♦♦ Free-Response Problems ♦♦♦

7. At baseball practice, 1498 pitches were thrown. If 7 players received the same number of pitches, how many pitches did each player receive? 214 pitches each

8. Mr. Park was born on October 10th, 1976. How old is Mr. Park today?

   45 years old

9. The smaller angle between the hour and minute hands of a clock at 9:00 pm is 90 degrees. At 7:20 pm, what is the smaller angle in degrees between the hour and minute hands on the clock?

   100 degrees

10. What is the ten's place digit of the following product? 1

$$\overbrace{11 \times 11 \times 11 \times 11 \times 11 \times \cdots \times 11 \times 11 \times 11 \times 11 \times 11}^{2021\ times}$$

Example for 5th Grade

♦♦♦ Multiple-Choice Problems ♦♦♦

11. Which of the following numbers is closest to 2021?

(a) 2020.899 (b) 2020.91 (c) 2021.1 (d) 2021.018 (e) 2021.03

12. James, Mary, Linda, and Joseph went on a road trip. The trip was 120 miles. James drove $\frac{1}{2}$ of the way, and Mary drove $\frac{1}{3}$ of the way. Linda and Joseph split the rest of the driving equally. How far did Linda drive?

(b) 5 miles (b) 10 miles (c) 15 miles (d) 20 miles (e) 25 miles

13. If a train departed Penn station at 11:45 a.m. and arrived at Cold Spring Harbor at 2:30 p.m., how long did the trip take?

(a) 2 hours 15 minutes (b) 2 hours 30 minutes (c) 2 hours 45 minutes

(d) 3 hours 15 minutes (e) 3 hours 30 minutes

14. Which of the following is equal to 19×76?

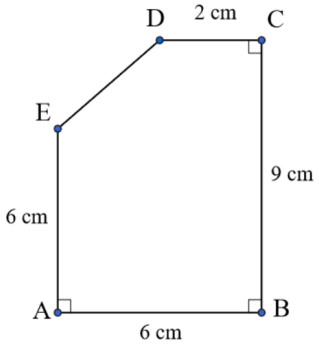
(a) $(1 \times 76) + (9 \times 76)$ (b) $(10 \times 76) + (9 \times 76)$ (c) $(10 \times 70) + (9 \times 6)$

(d) $(19 \times 7) + (19 \times 6)$ (e) $(10 \times 6) + (9 \times 70) + (9 \times 6)$

15. What is the value of $1 - 2 + 3 - 4 + 5 - 6 + \cdots + 2019 - 2020 + 2021 = ?$

(a) -1011 (b) -1010 (c) 0 (d) 1010 (e) 1011

16. In the figure below, AB = 6 cm, BC = 9 cm, CD = 2 cm, and AE = 6 cm. What is the area of the polygon ABCDE in square centimeters (cm^2)? (*Note: Figure not drawn to scale!*)



(a) 44 (b) 46 (c) 48 (d) 50 (e) 52

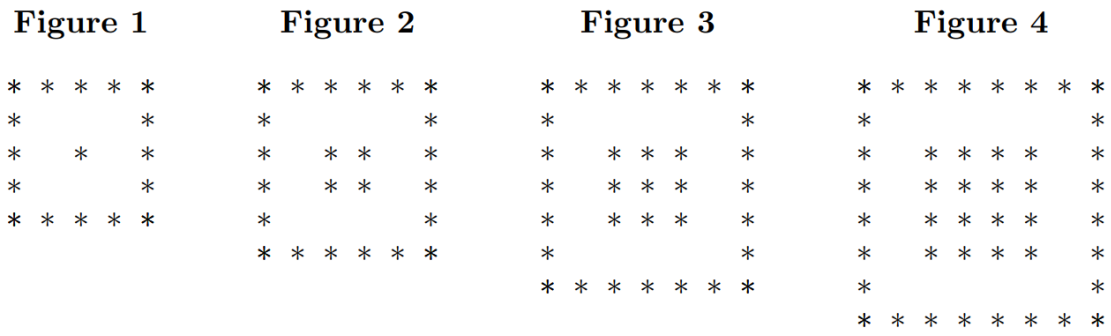
♦♦♦ Free-Response Problems ♦♦♦

17. The toy factory ships toys in crates that are 3 feet wide, 3 feet long, and 3 feet high. If each toy is 1 foot wide, 3 feet long, and 1 foot high, how many toys can go into each crate? 9 toys

18. James counted numbers backwards from 2021 by 7's: 2021, 2014, 2007, 2000, · · · . What is the smallest positive number he counted? 5

19. Julia scored the highest grade on the math test with a 95. Richard scored 5 points less than Julia. Tom received 80% of Richard's score. What did Tom score on his test? 72

20. If the pattern of stars in the following figures continues, how many stars would be in Figure 10? 152



Example for 6th Grade

♦♦♦ Multiple-Choice Problems ♦♦♦

21. Which is the smallest in the set $\{\frac{2}{3}, 25\%, \frac{2020}{2021}, 0.15, 70\%\}$?

(b) $\frac{2}{3}$ (b) 25% (c) $\frac{2020}{2021}$ (d) 0.15 (e) 70%

22. John's weight is 65 kilograms. One kilogram is equal to 2.203 pounds. Round this weight of 65 kilograms to the nearest pound.

(a) 143 (b) 145 (c) 147 (d) 149 (e) 151

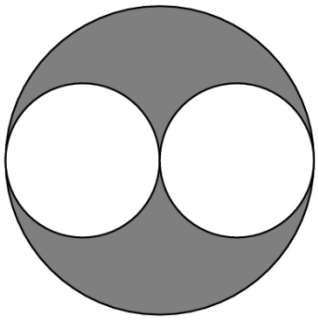
23. As a result of a survey of 175 New York tourists, 83 visited The Met, 77 visited MoMA, and 41 visited both museums last year. What percent of the people in the survey did not visit either museum last year?

(a) 8.5% (b) 23.4% (c) 32% (d) 36.5% (e) 38%

24. The taxi fare in New York City is initially \$2.50 plus an additional 50 cents per $\frac{1}{5}$ mile. You plan to give the driver a \$3 tip. How many miles can you ride for \$20?

(a) 4.8 (b) 5.8 (c) 6.8 (d) 7 (e) 7.8

25. In the diagram below, the diameter of each of the two smaller circles is the radius of the larger circle. If the area of the two small circles combined is 12 cm^2, what is the area of the shaded region? (*Note: Figure not drawn to scale!*)



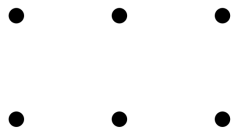
(a) 3 cm^2 (b) 4 cm^2 (c) 6 cm^2 (d) 12 cm^2 (e) 6π cm^2

26. Suppose a, b, and c are integers and $|a + 5| + |b - 8| + |c + 3| = 1$. Which of the following could be $|a + b + c|$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

♦♦♦ Free-Response Problems ♦♦♦

27. How many different triangles can be drawn by using any three of the 6 dots below as vertices of each triangle? 18 triangles



28. Suppose 12% of x equals 15% of y. What percentage of x is y? 80%

29. Jennifer has 11 coins in her pocket totaling $1.37, and she has at least one of every type. How many dimes are there? 2

30. Find the sum of all values of x which satisfy the following equation. $\frac{38}{5}$ or $7\frac{3}{5}$

$$\frac{x-2}{3}+\frac{x-3}{2}=\frac{3}{x-2}+\frac{2}{x-3}$$

Example for 7th Grade

♦♦♦ Multiple-Choice Problems ♦♦♦

31. Find the value of $2021\frac{1}{6} - \frac{2}{3} - 2015\frac{1}{2}$.

(c) 4 (b) $4\frac{1}{6}$ (c) $4\frac{1}{3}$ (d) $4\frac{1}{2}$ (e) 5

32. What is the sum of all valid solutions of the fractional equation $\frac{x}{x-1} = \frac{x+1}{1-x}$?

(a) $-\frac{1}{2}$ (b) 0 (c) $\frac{1}{2}$ (d) 1 (e) 2

33. Use the list of numbers below to answer the question that follows.

2, 5, 7, 11, 13, k

If the mean of the list is 8, what is the median of the list?

(a) 7 (b) 7.5 (c) 8 (d) 8.5 (e) 11

34. The chart shows the high and low temperature (°C) in New York city.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
High	4°	5°	10°	16°	22°	26°	29°	28°	24°	18°	12°	7°
Low	-3°	-2°	2°	7°	12°	17°	20°	20°	16°	10°	5°	0°

What is the average of the low temperature? (Round your answer to the nearest tenth of a degree)

(a) 8.7° (b) 10.4° (c) 14.5° (d) 16.8° (e) 18.5°

35. When the length of a rectangle increases by 30% and the width of the rectangle decreases by 20%, what will happen to the area of the rectangle?

(b) increase by 10% (b) increase by 4% (c) increase by 1%

(d) decrease by 10% (d) decrease by 4%

36. David rolls three distinct dice and gets a, b, and c. Find the probability of $3a + b + c = 17$.

(b) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{18}$ (d) $\frac{1}{36}$ (e) $\frac{5}{108}$

♦♦♦ Free-Response Problems ♦♦♦

37. How many integers between 1000 and 9999 have four distinct digits? 4536

38. A clock with two hands indicates that it is 1:48 PM. Assuming the hour hand moves continuously between 1 and 2, find the measure of the angle between the two hands. 126° (or 234°)

39. If x is a number such that $x^2 + x + 1 = 0$. Then find the numerical value of

$$\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \cdots + \left(x^{2021} + \frac{1}{x^{2021}}\right)^2$$

4040

40. An infinite decimal, 0.13579111315171921 · · · , can be constructed using consecutive odd numbers (1, 3, 5, 7, 9, 11, 13, …) side-by-side starting from 1. In which decimal place is the digit 0 of 2021 located? 3487th

Example for 8th Grade

♦♦♦ Multiple-Choice Problems ♦♦♦

41. If $20^{21} = 2^x 5^y$, what is $x + y$?

(b) 21 (b) 24 (c) 42 (d) 51 (e) 63

42. Which of the following numbers is the 3rd largest?

$$2\sqrt{5}, \quad \frac{4}{5}, \quad 2 + 2\sqrt{2}, \quad \frac{\sqrt{60}}{2}, \quad \frac{5}{3}$$

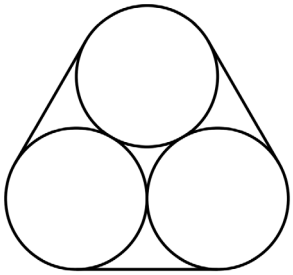
(a) $2\sqrt{5}$ (b) $\frac{4}{5}$ (c) $2 + 2\sqrt{2}$ (d) $\frac{\sqrt{60}}{2}$ (e) $\frac{5}{3}$

43. A cylindrical tank with a radius of 3 feet and a height of 30 feet is being filled with a liquid. There is already 2 feet of the liquid in the tank. If more liquid is poured into the tank at a rate of 18π (ft^3/minute) for 10 minutes, how high will the water be in the tank?

(b) $2 + \pi$ feet (b) $2 + 3\pi$ feet (c) $2 + 5\pi$ feet (d) 20 feet (e) 22 feet

44. A belt is drawn tightly around three circles of radius 10 cm each, as shown. The length of the belt, in cm, can be written in the form of $a + b\pi$, where a and b are integers. What is the value of $a - b$?

(a) 10 (b) 20 (c) 30 (d) 40 (e) 50



45. Let $\alpha + \beta = 2$ and $\alpha^2 + \beta^2 = 5$. Find the value of $\alpha^3 + \beta^3$.

(a) -5 (b) 2 (c) 5 (d) 8 (e) 11

46. What is the tens digit of the following sum?

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \cdots + 2019^2 - 2020^2 + 2021^2$$

(a) 0 (b) 1 (c) 3 (d) 5 (e) 7

♦♦♦ Free-Response Problems ♦♦♦

47. What is the value of x if $7^x + 7^x + 7^x + 7^x + 7^x + 7^x + 7^x = 7^{2021}$?

    2020

48. Write a linear equation to describe this relationship: 50°F = 10°C and –40°F = –40°C.

    $°F = \frac{9}{5}°C + 32$ or $°C = \frac{5}{9}(°F-32)$

49. Evaluate: $\sqrt{4^2 + 8^2 + 12^2 + 16^2 + \cdots + 36^2 + 40^2}$

    $4\sqrt{385}$

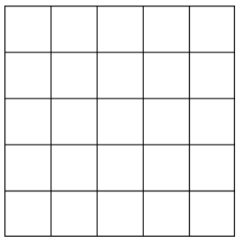
50. New York City is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 4 vans and 6 buses with 136 students. High School B rented and filled 2 vans and 4 buses with 86 students. Every van had the same number of students in it, and every bus had the same number of students. Find the number of students in each van and in each bus.

    Van: 7 and Bus: 18

Example for 9th Grade

♦♦♦ Multiple-Choice Problems ♦♦♦

51. How many squares are there of any size in this five-by-five grid of squares?



(c) 25 (b) 30 (c) 51 (d) 55 (e) 91

52. What is the value of $\frac{3^{2021}+3^{2019}}{3^{2021}-3^{2019}}$?

(a) $\frac{11}{9}$ (b) $\frac{13}{9}$ (c) $\frac{25}{9}$ (d) $\frac{5}{4}$ (e) $\frac{7}{4}$

53. How many ways can I arrange the six letters of the word ORANGE such that no vowel occupies an odd-numbered position?

(a) 36 (b) 72 (c) 108 (d) 324 (e) 720

54. The point $P(a, b)$ in the xy-plane is first rotated clockwise by 90° around the origin and then reflected about the line $y = -x$. The image of P after these two transformations is at (-5, 2). What is $a + b$?

(a) -10 (b) -7 (c) -3 (d) 7 (e) 10

55. Find the real number a that satisfies the identity:

$$a = 2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}$$

(a) $-\sqrt{2} - 1$ (b) $\sqrt{2} - 1$ (c) $\sqrt{2} + 1$ (d) $\frac{5}{2}$ (e) 3

56. There are 12 bills in the wallet: 5 ones, 3 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is $20 or more?

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{2}{11}$ (d) $\frac{10}{33}$ (e) $\frac{1}{3}$

♦♦♦ Free-Response Problems ♦♦♦

57. Find the integer n that satisfies the identity: 1010

$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{n}}} = \frac{3032}{2021}$$

58. In a pizza shop, it is possible to have on a pizza a choice of any combination of the following eight ingredients: 247

Pepperoni, Mushroom, Extra cheese, Sausage, Onion, Black olives, Green pepper, Chicken

How many different varieties of pizza can be offered if the pizza shop offers all pizzas with at least two toppings of any combination of the nine ingredients? *(It does not offer any one-topping pizza nor no-topping pizza.)*

59. Some positive integers are divisible by each of their digits. We call such a positive integer "digit divisible". For examples, 248 and 264 are digit divisible. Find the sum of all two-digit digit divisible integers. 630

60. How many positive integers less than or equal to 2021 are multiples of 3 or 4 but not 5? 808

Example for 10th Grade

♦♦♦ Multiple-Choice Problems ♦♦♦

61. Which of the following are in between $\frac{\sqrt{2}}{2}$ and $\frac{\sqrt{22}}{7}$?

(a) $\frac{\sqrt{12}}{5}$ (b) 0.7 (c) $\frac{\sqrt{17}}{6}$ (d) None of the above e) All of the above

62. What is the remainder when 2021×2022×2023 is divided by 9?

(a) 0 (b) 2 (c) 3 (d) 5 (e) 6

63. In Europe, dates are written as DD/MM, so today will be 20/11. Since there is no 20th month, it will not create any confusion. On the other hand, 10/4 is April 10th in Europe but October 4th in US, creating a confusion. How many days in a year would create confusion?

(a) 12 (b) 120 (c) 132 (d) 144 (e) 183

64. Palindromes are words which read the same backward as forward, and it works the same for numbers. For example, 1331 is a palindrome while 1133 is not. How many five-digit integers are palindromes?

(a) 729 (b) 810 (c) 900 (d) 999 (e) 1000

65. Troels was supposed to write 43210 on the board, but he mistakenly wrote down the same digit twice next to each other and created six-digit number instead. As a result, the new number is divisible by 7. What is the extra digit Troels put?

(a) 4 (b) 3 (c) 2 (d) 1 (e) 0

66. If it rains today, the probability of tomorrow raining is 3/4. If it doesn't rain today, the probability of tomorrow raining is 1/3. If it rained on Monday, what's the probability of raining on Wednesday of the same week?

(a) 1/2 (b) 7/12 (c) 31/48 (d) 11/16 (e) 12/13

♦♦♦ Free-Response Problems ♦♦♦

67. The (arithmetic) mean of $x - 1$, $2x + 3$, $4x + 7$, $3x + 2$, $6x + 1$ is 28. What is the median?

26

68. Eddie finishes the job twice as fast as Max. Max finishes the job three times as fast as Chris. When Eddie and Max worked together, it took 48 minutes. How long would it take if Max and Chris worked together? 108 minutes

69. There is a circle C_1. A square S_1 is inscribed in the circle C_1. Then another circle C_2 is inscribed in S_1, Similarly, S_2 is inscribed in C_2 and C_3 is inscribed in S_2. What is the area of C_1, if the area of C_3 is 1? 4

70. There are six letters; 'A', 'B', 'B', 'C', 'C', 'C'. How many 6-letter words can you make without 'ABC' in it? For example, 'BBACCC' is good while 'BABCCC' is not. 48

Example for 11th Grade

♦♦♦ Multiple-Choice Problems ♦♦♦

71. How many positive integers divide $(3!)^3$ evenly?

(a) 4 (b) 9 (c) 12 (d) 16 (e) 20

72. In how many ways can you go from (0, 0) to (5, 5) in 10 steps, where each step is (0, +1) or (+1, 0), without going to (2, 3)?

(a) 100 (b) 152 (c) 200 (d) 252 (e) 300

73. Suppose you have x and y and they are multiples of 3. How many from the following *can* be a multiple of 9?

$$(x+2)(y-1),\ \sqrt{xy},\ \frac{2x}{7y},\ \frac{(y+1)^2}{x}$$

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

74. There is a triangle and a rectangle. What is the maximum number of intersections, assuming no sides are overlapping?

(a) 3 (b) 4 (c) 6 (d) 7 (e) 8

75. There is a rectangular shaped lawn, 3 meters by 5 meters. Suppose (0, 0) represents the bottom left corner of the lawn. At (1m, 1m), there is a pole and a goat is tied to the pole with a 2-meter string. What's closest to the area that the goat can cover?

(a) 6 m^2 (b) 7 m^2 (c) 8 m^2 (d) 9 m^2 (e) 10 m^2

76. There are b marbles in a bag. Out of b, two are black. Erick put m more black marbles to the bag, and the probability to draw a black marble was increased 100%. If $0 < b < 10$, how many possible m are there?

(a) Not possible (b) 5 (c) 4 (d) 3 (e) 2

♦♦♦ Free-Response Problems ♦♦♦

77. Two sides of an isosceles triangle are 8 and 17. What is the area of the triangle? $4\sqrt{273}$

78. Stephen and Jong each tosses a fair coin three times. What is the probability that Stephen had more heads than Jong did? $11/32$

79. We have $x + y = 20$, $y + z = 28$. If x, y, and z form an arithmetic sequence, what is the sum of all possible x? 24

80. Suppose $a + b + c = 2$, $a^2 + b^2 + c^2 = 14$, and $a^3 + b^3 + c^3 = 20$. What is abc? -6